Problem 10

Use the preliminary test to decide whether the following series are divergent or require further testing. *Careful:* Do *not* say that a series is convergent; the preliminary test cannot decide this.

$$\sum_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)$$

Solution

Take the limit of the summand as $n \to \infty$.

$$\begin{aligned}\lim_{n\to\infty}\left(1-\frac{1}{n^2}\right) &= \lim_{n\to\infty} 1 - \lim_{n\to\infty}\frac{1}{n^2} \\ &= 1-0 \\ &= 1\end{aligned}$$

Since it's not zero, the series diverges by the preliminary test.